

# Shaping optimal transmitter waveforms for marine CSEM surveys

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## Summary

A much used transmitter waveform for marine CSEM surveys is the square wave. The square wave has the advantage that maximum energy is transferred to the subsurface, since the transmitter current is running at its peak amplitude at all times. The problem with the square wave is that it has a less than ideal frequency spectrum. The frequency domain current amplitudes are proportional to the inverse of frequency, so the amplitudes are reduced with increased frequency. At the same time the absorption of the electromagnetic field increases with frequency. We propose a waveform where the transmitter operate at its peak current at all times, but where the number of switching times within a period may be larger than two, which is the number of switching times per period for a square wave. The method is based on matching desired frequency spectra with spectra obtained from generalized square waves. This is an optimization problem that is solved with a Monte Carlo method. The end results are waveforms that can be used for an electric dipole transmitter and where the frequency spectra are close to predefined desired spectra.

## Introduction

Marine controlled-source electromagnetics (CSEM) as suggested by Cox et al. (1971) and Young and Cox (1981) have later been followed up by contributions from Constable (1990), Constable and Cox (1996), Yuan and Edwards (2000) and MacGregor et al. (2001). These were non-hydrocarbon related studies except for Yuan and Edwards who investigated marine gas hydrates. The application of the method to hydrocarbon exploration, also named Sea Bed Logging (SBL), is described by Eidesmo et al. (2002) and Ellingsrud et al. (2002). The use of the method for hydrocarbon exploration in ExxonMobil is discussed by Srnka et al. (2006).

A much used transmitter waveform is the square wave. The square wave has the advantage that maximum energy is transferred to the subsurface, since the source current is running at its peak value at all times except for possible switching intervals. The problem with the square wave is that the current amplitudes are proportional to the inverse of frequency, so the current amplitudes are reduced with increased frequency. At the same time the absorption of the electromagnetic field increases with frequency. It may be desirable to partially counter act the increased loss with frequency by distributing more power to higher frequencies than is possible with the square wave.

We propose a generalized square wave that combines max-

imum power with a desired frequency spectrum. The advantage is that more than a few frequencies with appreciable amplitude can be acquired in a single tow line. The maximum power is obtained by having an active source at maximum negative or positive current at all times. The method is developed with the assumption that there may be a short period of zero current in the transmitter when switching from a negative to a positive current direction or vice versa. This is typical for a thyristor based transmitter where the wait time is approximately 80 - 100 ms. This switching interval will be much smaller for a transistor based transmitter, possibly less than 5 ms.

Several advanced processing schemes, like depth migration and inversion, give improved results as the number of frequencies that can penetrate to large depths increase. With a standard square wave this can be achieved by towing the same line several times with different base frequencies. It is clear that there is a potential cost reduction if a richer frequency spectrum can be obtained from a single towline.

## Theory

We consider a periodic waveform for the transmitter current, where the current is allowed to switch direction  $M$  times during a period. For the standard square wave, the transmitter current switch sign two times during a period so we allow  $M$  to be equal to or larger than two. We can consider  $M$  to be even since we work with a periodic signal. The transmitter current,  $J(t)$ , is assumed periodic with a period  $T_0$ ,

$$J(t) = J(t + T_0). \quad (1)$$

The transmitter current can be represented by the following Fourier series,

$$J(t) = \sum_{n=0}^{\infty} J_n e^{-i\omega_n t}. \quad (2)$$

The angular frequency  $\omega_n$  is,  $\omega_n = n\omega_0$ , with,  $\omega_0 = 2\pi f_0 = 2\pi/T_0$ . The Fourier representation of the transmitter current is,

$$J_n = \frac{2}{T_0} \int_0^{T_0} dt J(t) e^{i\omega_n t}. \quad (3)$$

We split a period into a time-ordered sequence,  $S_M$ , of  $M + 1$  times,

$$S_M = \{t_0, \dots, t_{m-1}, t_m, t_{m+1}, \dots, t_M\}, \quad (4)$$

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such that  $t_0 = 0$ ,  $t_M = T_0$  and  $t_{m-1} \leq t_m$ . We assume that the time  $t_0$  belongs to the previous period. The times,  $t_m$ , in this sequence are then switching times at which the direction of the current change  $\pm 180$  degrees with respect to the transmitter orientation. The current amplitude is assumed to be at the peak value between switching times. In the derivation to follow we choose the current to be at the positive peak value on the first interval from  $t_0$  to  $t_1$ . The other possible option is to choose the current to be at the negative peak value on this interval. The difference is a 180 degrees phase shift of the current waveform. The resulting spectra from these two choices will be identical if all the switching times are identical. Some of the examples we show in the following are with with a negative peak value on the first interval. For practical use, one choice is just as good as the other. Let  $J_{max}$  be the maximum current for the transmitter, then the proposed transmitter waveform can be written,

$$J(t) = (-1)^{m-1} J_{max} \quad t_{m-1} < t \leq t_m. \quad (5)$$

The above equation must be modified slightly if a thyristor based transmitter is analyzed. The reason is that such a transmitter cannot switch instantaneously between positive and negative current directions. It will have a switching interval,  $t_s$ , which is approximately 80 - 100 ms. In this interval the current is zero. We can include this effect by modifying equation 5. Let  $t_\sigma$  be half the switching interval, thus,

$$t_s = 2t_\sigma, \quad (6)$$

then the transmitter time function, including the effect of a finite switching interval, is,

$$\begin{aligned} J(t) &= 0 & t_{m-1} < t \leq t_{m-1} + t_\sigma, \\ J(t) &= (-1)^{m-1} J_{max} & t_{m-1} + t_\sigma < t \leq t_m - t_\sigma, \\ J(t) &= 0 & t_m - t_\sigma < t \leq t_m. \end{aligned} \quad (7)$$

The Fourier representation for the part of the transmitter current that is defined on the interval  $m$  is,

$$J_n(m) = J_{max} \frac{2}{T_0} (-1)^{m-1} \int_{t_{m-1}+t_\sigma}^{t_m-t_\sigma} dt e^{i\omega_n t}. \quad (8)$$

The integral gives,

$$J_n(m) = J_{max} \frac{2}{n\pi} (-1)^{m-1} e^{i\omega_n \tau_m} \sin[\omega_n (\Delta t_m - t_s)], \quad (9)$$

where,  $\tau_m = \frac{1}{2}(t_m + t_{m-1})$  and  $\Delta t_m = \frac{1}{2}(t_m - t_{m-1})$ .

The contribution from all intervals must be summed,

$$J_n = \sum_{m=1}^M J_n(m), \quad (10)$$

which with equation 9 give,

$$J_n = J_{max} \frac{2}{n\pi} \sum_{m=1}^M (-1)^{m-1} e^{i\omega_n \tau_m} \sin[\omega_n (\Delta t_m - t_s)]. \quad (11)$$

Equation 11 is our forward model for calculating the spectrum due to a given switching time sequence,  $S_M$ .

The spectrum of the standard square wave can be obtained by using  $M = 2$  and  $t_s = 0$  in equation 11,

$$|J_n| = J_{max} \frac{4}{n\pi} \left[ \frac{1}{2} (1 - (-1)^n) \right]. \quad (12)$$

This is a well known expression. It is only the odd harmonics that contribute to the signal and the amplitude of the first harmonic is a factor  $\frac{4}{\pi}$  higher than the transmitter maximum current. A realistic square wave must include the effect of the switching intervals,

$$|J_n| = J_{max} \frac{4}{n\pi} \left[ \frac{1}{2} (1 - (-1)^n) \right] \left| \sin\left[\omega_n \frac{1}{4}(T_0 - 2t_s)\right] \right|. \quad (13)$$

Equipped with a forward model (equation 11) that calculate the frequency spectrum from the switching time sequence, the next step is to match the spectrum  $|J_n|$  with a desired spectrum,  $I_n$ . This is an optimization problem with the switching time sequence being the unknown. To proceed we define the desired current spectrum,  $I_n$ , such that all amplitudes add up to 100 percent. As an example, if we target the second, third and fifth harmonic with 34 percent on the second harmonic and 33 percent on the third and fifth harmonic,  $I_n$  is given by,

$$\begin{aligned} I_2 &= 34, \\ I_3 &= 33, \\ I_5 &= 33. \end{aligned} \quad (14)$$

All other values of  $I_n$  are set to zero. The spectrum  $|J_n|$  will depend on the switching time sequence,  $S_M$ . The problem at hand is to find a switching time sequence,  $S_M$ , such that  $|J_n|$  has a distribution of amplitudes as a function of harmonics that is as close to  $I_n$  as possible. One possible criterion is to minimize the least square error,  $\epsilon$ ,

$$\epsilon = \sum_{n=0}^N (|J_n| - aI_n)^2. \quad (15)$$

The scale factor  $a$  is introduced so that the amplitudes of  $|J_n|$  and  $I_n$  are comparable. An estimate for the scale factor is given by the value that minimize equation 15 for a given time sequence  $S_M$ ,

$$a = \frac{\sum_{n=0}^N |J_n| I_n}{\sum_{n=0}^N I_n^2}. \quad (16)$$

The maximum frequency,  $f_{max}$ , used in the optimization determine the highest harmonic,  $N$ , to use in equation 16. We normally use a maximum frequency of 15 Hz since higher frequencies are hard to utilize in an SBL survey, accordingly,  $N\omega_0 = 2\pi f_{max}$ , or  $N = T_0 f_{max}$ .

At the outset we do not know the optimum value of  $M$ . Equation 15 must be minimized for a range of  $M$  values

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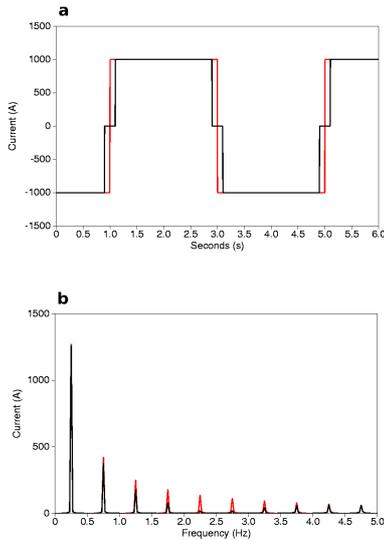


Fig. 1: Square waves with (black) and without (red) a switching time interval.

starting with  $M = 2$ . The maximum value of  $M$  will vary with the complexity of the problem, however there is clearly an upper limit when the switching interval,  $t_s$ , has a finite value. The total time the source is active during one period,  $T_{Eff}$ , can be written,

$$T_{Eff} = T_0 - Mt_s. \quad (17)$$

We see that if  $T_0 = 4$  s and  $M = 10$  then  $T_{Eff} = 3$  s for  $t_s = 0.1$  s. Hence, in the case of a finite  $t_s$ , the power output is reduced when  $M$  increase.

Equation 15 can in principle be solved by a local optimization scheme. However, we have chosen a global optimization scheme for this problem. We know that equation 15 has a multitude of local minima. The number of local minima increase with the number of switching times,  $M$ . Some of these local minima represent transmitter waveforms that are time shifted with respect to other waveforms in the solution space. These represent degenerate solutions and one solution is just as good as another solution. However, there are also local minima which represent distinctly different current spectra. There is also a global minimum for one of the values of  $M$ . Actually, there can be more than one “global minimum” for this problem. The reason is the degeneracy that is mentioned previously and which reflects the fact that solutions that differs by a simple phase shift only, are equally good with respect to the error functional and also with respect to practical use.

The calculation of the spectral contributions  $|J_n|$  is very fast so a Monte Carlo simulation is possible. We then have a scheme which have the potential of avoiding local minima. The scheme is as follows: The outer loop is over  $M$  which vary from 2 to 20, unless we find the smallest

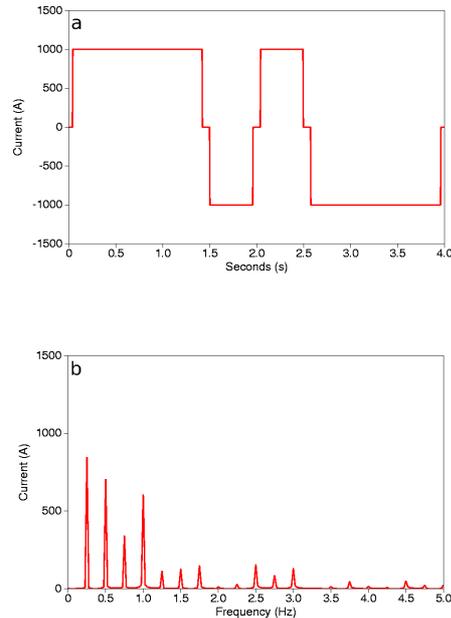


Fig. 2: Generalized square wave distributing currents mainly to the first, second and fourth harmonic.

error for  $M > 14$ . In this case the maximum value of  $M$  is increased further and we make sure that the smallest error is found for a value of  $M$  that is sufficiently small compared to the maximum value of  $M$  investigated. A margin of 6 as indicated here is a good choice by our experience. For fixed  $M$  we vary the elements of  $S_M$  using a random number generator. We calculate the error,  $\epsilon$ , and compare with previous values. We store  $S_M$  if the error is the smallest so far. For all values of  $M$  we let the number of iterations increase with a factor proportional to  $M^2$ , using  $M^2 \times 10^5$  iterations for  $M = 2$ . We get excellent results with less iterations, but use a large margin since the CPU time used to solve the problem is of no importance. It is perfectly acceptable to spend 10 CPU minutes for the calculation of a quantity that potentially will be used for many SBL surveys.

## Results

Figure 1 demonstrate the effect of a finite switching interval. The red curve in Figure 1a is the transmitter time-domain signal for an ideal square wave with period 4 seconds. The current is running at an absolute value of 1000 A. The spectrum is shown as the red curve in Figure 1b. The black curve in Figure 1a is the transmitter time domain signal for a square wave with a switching interval of 100 ms. The spectrum is shown as the black curve in Figure 1b. The spectra here and in the following are calculated by repeating 10 periods of the time domain signal

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before the Fourier transforms are performed. From Figure 1b we observe that the effect of the finite switching intervals is small on the first harmonic, but we see a reduction of amplitude with frequency. This is an expected effect since there must be reduced current amplitudes in the spectrum due to the finite switching interval as can be seen from equation 17.

Figure 2 show a solution to a design problem where we require,  $I_1 = 34$ ,  $I_2 = 33$  and  $I_4 = 33$ . At the same time we require that there is no DC component in the transmitter current spectrum. The switching interval is 80 ms and no solution is accepted unless all intervals with an active source are larger than 200 ms. Figure 2a is the transmitter time domain signal for a generalized square wave with a period of 4 seconds. The best solution is found for  $M = 4$ . The spectrum is shown in Figure 2b. All the desired current amplitudes are larger than 600 A.

A real data example is shown in Figures 3 and 4. Figure 3a is the recorded transmitter time domain signal for a generalized square wave with period 8 seconds. The corresponding spectrum is shown in Figure 3b. The distribution of current amplitudes having 200 A or more are,

$$\begin{aligned}
 J_x(0.125 \text{ Hz}) &= 240A, \\
 J_x(0.250 \text{ Hz}) &= 750A, \\
 J_x(0.375 \text{ Hz}) &= 520A, \\
 J_x(0.500 \text{ Hz}) &= 480A, \\
 J_x(0.625 \text{ Hz}) &= 370A, \\
 J_x(1.250 \text{ Hz}) &= 200A.
 \end{aligned}
 \tag{18}$$

The inline electric receiver data for these frequencies are shown in Figure 4. The color coding is,

$$\begin{aligned}
 E_x(0.125 \text{ Hz}) &= \text{black}, \\
 E_x(0.250 \text{ Hz}) &= \text{red}, \\
 E_x(0.375 \text{ Hz}) &= \text{blue}, \\
 E_x(0.500 \text{ Hz}) &= \text{green}, \\
 E_x(0.625 \text{ Hz}) &= \text{grey}, \\
 E_x(1.250 \text{ Hz}) &= \text{cyan}.
 \end{aligned}
 \tag{19}$$

The data clearly demonstrate the frequency dependent absorption of marine electromagnetic data. The strongest transmitter current is for the frequency of 0.25 Hz and the electric field amplitude for this frequency is the largest up to approximately 8 km. For higher offsets the electric field amplitude for 0.125 Hz is the largest even if the current amplitude is less than 1/3 of the 0.25 Hz current amplitude. This is due to less absorption at this frequency. The current amplitudes at 0.125 Hz and at 1.25 Hz are of similar size. Comparing the electric fields at these frequencies we see that the 1.25 Hz data stay above the noise floor up to 5 km whereas the 0.125 Hz data stay above the noise floor for all offsets. These data are recorded with the EMGS generation I receivers. The noise floor for the generations II receivers is one order of magnitude lower, this makes it possible to utilize high frequency data to even higher offsets.

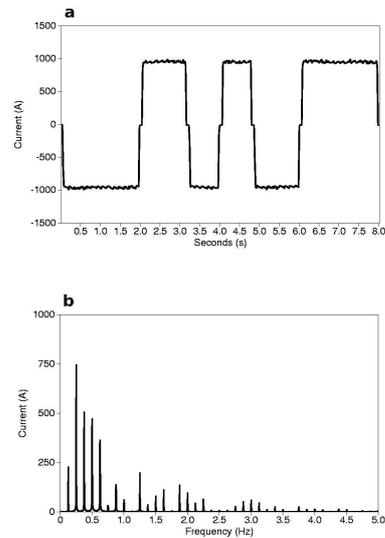


Fig. 3: Recorded transmitter waveform with period 8 seconds.

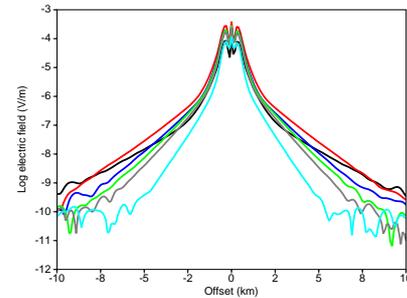


Fig. 4: Recorded inline electric field due to waveform in Figure 3.

## Conclusion

We have introduced a method for generating transmitter waveforms that give the possibility to adapt the frequency distribution of the current amplitudes to a desired distribution. The method is designed to maximize the time the source is switched on at maximum power. We observe that by introducing this generalized square wave, we have a large freedom in distributing current amplitudes between different frequencies. In contrast to the standard square wave we can have relatively large currents on both even and odd harmonics. We have also tested the method in data acquisition and demonstrated that recorded signals behaves according to what we expect from the theory.

## EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2007 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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